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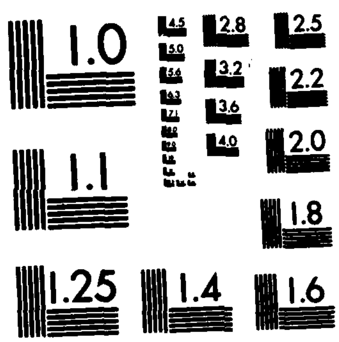

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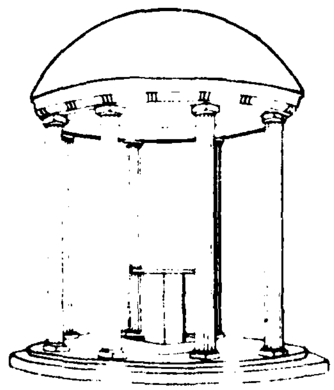
EVALUATION OF A "LARGE POP" FILTERING RULE FOR INVENTORY MANAGEMENT SYSTEMS

Technical Report #22

Douglas Blazer

February 1983



SCHOOL OF BUSINESS ADMINISTRATION AND CURRICULUM IN OPERATIONS RESEARCH AND SYSTEMS ANALYSIS

EVALUATION OF A "LARGE POP" FILTERING RULE
FOR INVENTORY MANAGEMENT SYSTEMS

Technical Report #22

Douglas Blazer

February 1983

Work Sponsored By

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Decision Control Models in Operations Research

Harvey M. Wagner
Principal Investigator
School of Business Administration
University of North Carolina at Chapel Hill

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In this report we evaluate the performance of an inventory filtering rule on computer simulated individual customer orders. The filtering rule identifies a threshold value for which all orders exceeding that value are not filled from existing stock, but rather are specially handled. We present classical statistical outlier rules and other inventory filtering rules and show how they do not perform well in a practical inventory setting. We develop an inventory filtering rule, and test its performance on seven

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different customer order distributions that resemble distributions we have seen in practice. We show that for practical inventory applications, our filtering rule statistically outperforms other models currently in the literature.

FOREWARD

As part of the on-going program in "Decision Control Models in Operations Research," Mr. Douglas Blazer has extended the study of removing large demands in the determination of an inventory replenishment policy. In this report, he describes an inventory filtering rule which identifies a threshold value T such that any order equal to or exceeding T is specially handled. He compares the statistical performance of this filtering rule to the performance of classical statistical outlier rules and other inventory filtering rules. The paper provides results on 33 cases using 7 different customer order distributions. Other related reports dealing with this research program are given on the following pages.

Harvey M. Wagner
Principal Investigator

Richard Ehrhardt
Co-Principal Investigator

Ehrhardt, R. (1977), Operating Characteristic Approximations for the Analysis of (s,S) Inventory Systems, ONR and ARO Technical Report 12, April 1977, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 109 pp.

Schultz, C. R., R. Ehrhardt, and A. MacCormick (1977), Forecasting Operating Characteristics of (s,S) Inventory Systems, ONR and ARO Technical Report 13, December 1977, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 47 pp.

Schultz, C. R. (1979), (s,S) Inventory Policies for a Wholesale Warehouse Inventory System, ONR Technical Report 14, April 1979, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 75 pp.

Schultz, C. R. (1980), Wholesale Warehouse Inventory Control with Statistical Demand Information, ONR Technical Report 15, December 1980, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 74 pp.

Ehrhardt, R. and G. Kastner (1980), An Empirical Comparison of Two Approximately Optimal (s,S) Inventory Policies, Technical Report 16, December 1980, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 22 pp.

Ehrhardt, R. (1980), (s,S) Policies for a Dynamic Inventory Model with Stochastic Lead Times, Technical Report 17, December 1980, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 20 pp.

Mosier, C. (1981), Revised (s,S) Power Approximation, Technical Report 18, February 1981, School of Business Administration, University of North Carolina at Chapel Hill, 18 pp.

Blazer, D. and M. McClelland (1981), An Inventory Model for Special Handling of Extreme Value Demands, Technical Report 19, December 1981, School of Business Administration, University of North Carolina at Chapel Hill, 10 pp.

Mitchell, J. (1982), Choosing Single-Item Service Objectives in a Multi-Item Base-Stock Inventory System, Technical Report 20, School of Business Administration, University of North Carolina at Chapel Hill, 30 pp.

MacCormick, A. (1974), *Statistical Problems in Inventory Control*, ONR and ARO Technical Report 2, December 1974, School of Organization and Management, Yale University, 244 pp.

Estey, A. S. and R. L. Kaufman (1975), *Multi-Item Inventory System Policies Using Statistical Estimates: Negative Binomial Demands (Variance/Mean = 9)*, ONR and ARO Technical Report 3, September 1975, School of Organization and Management, Yale University, 85 pp.

Ehrhardt, R. (1975), *Variance Reduction Techniques for an Inventory Simulation*, ONR and ARO Technical Report 4, September 1975, School of Organization and Management, Yale University, 24 pp.

Kaufman, R. (1976), *Computer Programs for (s,S) Policies Under Independent or Filtered Demands*, ONR and ARO Technical Report 5, School of Organization and Management, Yale University, 65 pp.

Kaufman, R. and J. Klincewicz (1976), *Multi-Item Inventory System Policies Using Statistical Estimates: Sporadic Demands (Variance/Mean = 9)*, ONR and ARO Technical Report 6, School of Organization and Management, Yale University, 58 pp.

Ehrhardt, R. (1976), *The Power Approximation: Inventory Policies Based on Limited Demand Information*, ONR and ARO Technical Report 7, June 1976, School of Organization and Management, Yale University, 106 pp.

Klincewicz, J. G. (1976), *Biased Variance Estimators for Statistical Inventory Policies*, ONR and ARO Technical Report 8, August 1976, School of Organization and Management, Yale University, 24 pp.

Klincewicz, J. G. (1976), *Inventory Control Using Statistical Estimates: The Power Approximation and Sporadic Demands (Variance/Mean = 9)*, ONR and ARO Technical Report 9, November 1976, School of Organization and Management, Yale University, 52 pp.

Klincewicz, J. R. (1976), *The Power Approximation: Control of Multi-Item Inventory Systems with Constant Standard-Deviation-To-Mean Ratio for Demand*, ONR and ARO Technical Report 10, November 1976, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 47 pp.

Kaufman, R. L. (1977), *(s,S) Inventory Policies in a Nonstationary Demand Environment*, ONR and ARO Technical Report 11, April 1977, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 155 pp.

Blazer, D. (1983), Operating Characteristics for an Inventory Model That Special Handles Extreme Value Demand, Technical Report #21, School of Business Administration, University of North Carolina at Chapel Hill, 15 pp.

ABSTRACT

In this report we evaluate the performance of an inventory filtering rule on computer simulated individual customer orders. The filtering rule identifies a threshold value for which all customer orders exceeding that value are not filled from existing stock, but rather are specially handled. We show how classical statistical outlier rules and other inventory filtering rules do not perform well in a practical inventory setting. We develop an inventory filtering rule, and test its performance on seven different customer order distributions that resemble distributions we have seen in practice. We show that for practical inventory applications, our filtering rule statistically outperforms other models currently in the literature.

TABLE OF CONTENTS

1. INTRODUCTION

We showed in [2], [3] that cost savings can result from excluding extreme value demands in a single-item periodic review inventory system. We assumed that any week's demand exceeding the value τ is specially handled. We investigated values of τ that excluded the upper 5 to 15% of the cumulative probability of demand. We found that the cost savings from reduced inventory are often greater than the cost of special handling.

In a real situation, however, the probability distribution of a week's demand is not known exactly. Furthermore, an inventory manager cannot collect individual orders during a week in order to determine if the week's total demand exceeds the threshold value. Practical applications require a special handling rule that is based on historical observations of individual customer orders. The rule must specify a value T such that any individual order exceeding T is specially handled. Filtering out these "large pops" should result in a probability distribution of weekly demand that resembles the truncated demand distributions of [2].

In this report, we test a particular filtering rule using computer simulated individual customer orders. The order distributions that we postulate bear close resemblance to those that we have seen in actual data. We examine, in Section 2, the weaknesses of standard statistical outlier formulas for practical inventory

applications. In Section 3, we describe the basic filtering rule. Then, in Section 4, we describe the experimental design used to test the filtering rule. In Section 5 we display the results using the filtering rule as well as two other rules currently in the literature. We examine the operating characteristics for the filtering rule using different parameter settings. In Technical Report 23, we use empirical data to examine the cost effectiveness of the filtering rule under different parameter settings.

2. STATISTICAL OUTLIER THEORY

Statistical outlier theory has not had widespread use in an inventory context. What causes difficulty is that an outlier in one context may not be considered such in other contexts. Consider the following definition [1]:

> ...an outlier in a set of data is an observation (or subset of observations) which appears to be inconsistent with the remainder of the set of data. The phrase "appears to be inconsistent" is crucial. It is a matter of subjective judgement on the part of the observer whether or not he picks out some observation (or set of observations) for scrutiny.

Sometimes outliers are defined to include "errors of observation or misrecordings" and only when such values can be proven to be in error. Clearly, large demands that occur in a practical inventory setting are not "errors or misrecordings". Nonetheless, they could be considered outliers, at least in the sense that they can be handled differently to save inventory investment costs. Before we examine further the weaknesses of standard statistical outlier models in an inventory context, we group the models into four classes. Three examples are provided to illustrate the weakness of these statistical models in

an inventory setting.

2.1 STATISTICAL OUTLIER MODEL CLASSIFICATION

Classification is according to the form of the alternative hypothesis [1].

1. The first classification is deterministic, and covers outliers caused by reading or recording errors. If any observation is clearly in error, the basic hypothesis that all observations come from a single distribution, say F, is rejected. Thus, "rejection of the initial model in favor of the alternative is deterministically correct" [1].

2. The second classification, inherent variability models, examines the "possibility that outliers have appeared in the data as a result of a greater degree of inherent variability" in the distribution [1]. Hence the hypothesis that all observations are from some distribution, say F (H:F), is countered by the hypothesis that the observations actually arise from a different distribution we call G (H:G).

3. Mixture models test whether a few members of a sample arise from a population different from that represented by the basic model. Hence, there is a mixture of two population distributions in the sample, with the few members possibly being revealed as outliers. The appropriate hypothesis would be:

$$H:F \text{ versus } H:(1-\phi)F + \phi G$$

where $1-\phi$ is the proportion of demands arising from the distribution F and ϕ is the proportion of demands arising from the distribution G [1].

4. The final classification is slippage models. In these models most observations are derived from a population distribution F with parameters μ and σ, while some small number of observations arise independently from a modified version of F where either μ or σ have changed in value. Hence, the null hypothesis is that all observations arise from distribution F($H:F(\mu,\sigma)$), and the alternative hypothesis is that at least one observation arises from a modified version of F, either $H:F(\mu,b\sigma)$ or $H:F(\mu+a,\sigma)$.

In an inventory context, all four types of models appear to be plausible with the mixture and slippage models seeming to be the most likely. There are problems, however, arising from the use of any of these statistical models.

2.2 WEAKNESSES OF STATISTICAL MODELS

Many of the models in all four classifications imply that an outlier is an error, and therefore an unusual occurence. This view supports the assumption that a set of data contains at most one outlier [7]. But a model that tests only the most extreme value in a data set is not practical in an inventory setting, because multiple extreme values frequently exist. Models that test for multiple outliers use either consecutive (sequential) or block procedures [1]; these also have inherent weaknesses for use in an inventory management context.

Consecutive (or sequential) testing "implies that the sample size is not fixed, but is determined in each realization in relation to the values of the earlier observations."[1] For example, the test statistic by Likes [11], Kabe [10], is $\frac{(X_n-X_{n-1})}{(X_n-X_1)}$, where X_1 is the value of the

smallest observation and X_n is the value of the largest observation.

Assume sample values are 7, 7, 8, 9, 10, 951, 952. Consecutive testing would indicate that X_n is not an outlier and the procedure is terminated. The illustrative sample is not atypical in an inventory setting where there may even be several ties at an extreme value. This effect of several larger values is termed masking and is an inherent weakness in many statistical consecutive testing techniques [12].

Block procedures suffer from an alternative danger called swamping [5]. Blocking procedures test a group of outliers at one time, "en bloc". For example, the test statistic for multiple outliers by Fieller is

$$\frac{(X_{n-k+1}) + \ldots + X_n}{\sum_{j=1}^{n} X_j},$$

where k upper outliers are tested as a group. Assume sample values are 7, 7, 8, 9, 10, 951. Block procedures for k=2 indicate that both 10 and 951 are outliers, thus 10 is "swamped" by the value 951 [1].

Another weakness is that most classical statistical models are tailored to specific distributions. Barnett and Lewis list nearly 60 tests for outliers in univariate samples which apply to individual distributions (for example, Normal, Gamma, and Exponential). They report that, "...the performance of some of the classical tests or estimates is very unstable under small changes of the underlying distribution..." [9], [1]. In an inventory context, the individual order distribution is usually not known. In fact, individual order distributions

may be different for different items (or classes of items). Thus, tests for specific distributions are not practical for inventory uses. The fact that the underlying individual order distributions are unknown virtually excludes the use of the inherent variability, mixture, and slippage models.

2.3 EXAMPLES OF STATISTICAL MODELS

In Table 1 we provide three examples of customer order data to illustrate the effectiveness of seven statistical outlier models. Table 2 provides the statistical outlier models and a short description of their properties. Tables 3, 4, and 5 provide the results of applying the seven statistical models to the three order samples. The tables list the appropriate test statistic, with a dashed line to indicate which values are considered outliers. The single and double upper outlier models are applied sequentially, so that the first observation is excluded from consideration in the second application of the test.

The tables highlight the weaknesses of statistical outlier models. Single upper outlier tests apply best when an outlier is assumed to be an error of observation and therefore rare. They are not meant to be applied sequentially, and they give unreliable results, as in example A where three of the models (1, 3, and 4) consider all the observations to be outliers. Masking effects are evident in examples B and C for models 2 and 4. Model 7 is unreliable as the number of observations increases, and is useful only for small k (note the critical value formulation). Model 5 appears reliable; however, tables are available only for k=4. It appears for example C that more observations would be considered outliers had statistical tables been available; for k=1

there are no outliers, and for K=2 there are no outliers at the .01 significance level. Models 5 and 6 also have a disadvantage in the amount of data that must be collected to apply the models. In an inventory setting with 10,000 or more line items, the data storage requirements could be impractical.

TABLE 1

Example Data

List	A	B	C
Number in Sample	17	15	25
Median	30	500	150
Mean	632	1256	325
Standard Deviation	1313.3	1596	356
Observations	5000	5000	1200
	2500	5000	1000
	1358	2000	1000
	1000	1000	1000
	500	1000	500
	175	1000	500
	50	1000	450
	50	500	400
	30	500	315
	15	500	226
	15	500	226
	10	480	200
	10	300	200
	10	50	150
	10	15	150
	6		150
	2		108
			100
			100
			50
			50
			36
			10
			10
			5

TABLE 2

Statistical Outlier Models

Type Model	Model	Properties
Single Upper Outlier Models	$\frac{X_{(n)}-\bar{X}}{s}$	Where: $(X_n) \equiv$ single upper outlier value $\bar{X} \equiv$ mean demand $s \equiv$ standard deviation of demand (includes all observations). This model is a location slippage alternative test for a single upper outlier. The test is used to identify a single upper outlier in a normal sample. [1]
	$\frac{X_{(n)}-X_{(n-1)}}{X_{(n)}-X_{(1)}}$	Where: $X_{(n)} - X_{(n-1)} \equiv$ difference in the first two order statistics $X_{(n)} - X_{(1)} \equiv$ range. This model is a location slippage alternative test for a single outlier. The test is vulnerable to the masking effect and can be used for exponential, gamma, and normal sample distributions. [1]
	$\frac{X_{(n)}-\bar{X}}{s_v}$	Where: $s_v \equiv$ standard deviation of demand for the sample excluding X_n. Note this is a slight variation from s_v being an independent estimate of the standard deviation with v degrees of freedom. Again, the test is for location slippage alternative from a normally distributed sample. [1]

TABLE 2 (Continued) Statistical Outlier Models

Type Model	Model	Properties
Two Upper Outlier Models	$\frac{X_{(n)}-X_{(n-2)}}{X_{(n)}-X_{(1)}}$	This test can be used also for a single outlier to avoid the masking effect (or ties in extreme value observations). The test is used for gamma, exponential and normal samples. [1]
k Upper Outlier Models	$\frac{X_{(n-k+1)}+\ldots+X_n-k\bar{X}}{s}$	Where: $X_{(n-k+1)}+\ldots+X_{(n)}$ ≡ sum of the k upper observation values. The test is for location slippage alternative for k observations from a normal distribution. [1]
	$\frac{S^2_{n-k+1,\ldots,n-1,n}}{S^2}$	Where: $S^2_{n-k+1,\ldots,n-1,n}$ ≡ sum of squares excluding the $n-k+1,\ldots,n-1,n$ th value observations. S^2 ≡ total sum of squares (includes all observations). This model is a location slippage alternative test for k observations from a normal sample. [1]
	$\frac{X_n+\ldots+X_{(n-k+1)}}{\sum_j X_j}$	Where: $\sum_j X_j$ ≡ sum of all observations. The test is used for gamma or exponential samples. The best results are when the number of outliers, k, is small. [1]

TABLE 3

Statistical Outlier Models Test Statistics

<u>Example Data A</u>

Model Number	1	2	3	4	5	6	7
Observation Value	$\frac{X_{(n)}-\bar{X}}{s}$	$\frac{X_{(n)}-X_{(n-1)}}{X_{(n)}-X_{(1)}}$	$\frac{X_{(n)}-\bar{X}}{s_v}$	$\frac{X_{(n)}-X_{(n-2)}}{X_{(n)}-X_{(1)}}$	$\frac{X_{(N-k+1)}+\ldots+X_n-k\bar{X}}{s}$	$\frac{S^2_{n-k+1,\ldots,n-1,n}}{s^2}$*	$\frac{X_{(n)}+\ldots+X_{(n-k+1)}}{\Sigma X_j}$***
5000	3.33	.500	6.64	.729	3.33	-	.460
2500	3.06	.457	5.48	.600	4.75	.182	.698
1358	2.74	.264	4.37	.633	5.30	.663	.822
1000	3.07	.500	6.77	.826	5.58	.158	.916
500	3.14	.650	9.77	.904			.960
175	2.99	.722	9.23	.722			
50				.417			
Critical Value** (α=.05)	2.47	.321	2.88	.407	4.96 (k=3)	.239 (k=4)	

All values above the dashed line (– – – –) are considered outliers.

NOTES

*Tables were available for $k \le 4$ only. Test is for values less than the critical value.

**Extracted from tables in Barnett and Lewis.

***Table not available: Critical Value $\le \binom{n}{k} P\left(F_{2kr,2(n-k)r} > \frac{(n-k)t}{k(1-t)}\right)$

where $r \equiv$ gamma distribution shape parameter and $t \equiv$ test statistic value.

TABLE 4

Statistical Outlier Models Test Statistics

Example Data B

Model Number	1	2	3	4	5	6	7
	$\frac{X_{(n)}-\bar{X}}{s}$	$\frac{X_{(n)}-X_{(n-1)}}{X_{(n)}-X_{(1)}}$	$\frac{X_{(n)}-\bar{X}}{s_v}$	$\frac{X_{(n)}-X_{(n-2)}}{X_{(n)}-X_{(1)}}$	$\frac{X_{(n-k+1)}+\ldots+X_n-k\bar{X}}{s}$	$\frac{S^2_{n-k+1,\ldots,n-1,n}}{s^2}$ *	$\frac{X_{(n)}+\ldots+X_{(n-k+1)}}{\Sigma X_j}$ ***
Observation Value							
5000	2.34	0	3.18	.600	2.34	-	.265
5000	3.18	.600	8.23	.800	4.69	.214	.530
2000	2.51	.504	3.98	.504	5.16	.108	.636
1000	1.20	0	1.34	0	4.99	.107	.688
1000		0	1.57	0		.106	.740
1000		0	1.96	.507		.105	.798
1000		.507	3.05			.104	.847
Critical Value** (α=.05)	2.41	.339	2.88	.422	4.71 (k=3)	.196 (k=4)	

All values above the dashed line (– – – –) are considered outliers.

NOTES

*Tables were available for $k \le 4$ only. Test is for values less than the critical value.

**Extracted from tables in Barnett and Lewis.

***Table not available: Critical Value $\le \binom{n}{k} P\left(F_{2kr,2(n-k)r} > \frac{(n-k)t}{k(1-t)}\right)$

where $r \equiv$ gamma distribution shape parameter and $t \equiv$ test statistic value.

TABLE 5

Statistical Outlier Models Test Statistics

Example Data C

Model Number	1	2	3	4	5	6	7
Observation Value	$\frac{X_{(n)}-\bar{X}}{s}$	$\frac{X_{(n)}-X_{(n-1)}}{X_{(n)}-X_{(1)}}$	$\frac{X_{(n)}-\bar{X}}{s_v}$	$\frac{X_{(n)}-X_{(n-2)}}{X_{(n)}-X_{(1)}}$	$\frac{X_{(n-k+1)}+\ldots+X_n-k\bar{X}}{s}$	$\frac{S^2_{n-k+1,\ldots,n-1,n}}{s^2}$ *	$\frac{X_{(n)}+\ldots+X_{(n-k+1)}}{\Sigma X_j}$ ***
1200	2.46	.167	2.92	.167	2.46	-	.147
1000	2.28	0	3.33	0	4.35	.60	.270
1000	3.33	0	5.08	.502	6.24	.45	.393
1000	5.08	.502	5.64	.502	8.14	.30	.516
500	2.24	0	2.70	.101	8.63	.29	.575
500	2.70	.101	3.30		9.12	.28	.636
450	2.15						
Critical Value** (α=.05)	2.65	.277	2.74	.343	4.33 (k=2) 6.74 (k=4)	.54 (k=2)	

All values above the dashed line (– – – –) are considered outliers.

NOTES

*Tables were available for $k \le 4$ only. Test is for values less than the critical value.

**Extracted from tables in Barnett and Lewis.

***Table not available: Critical Value $< \binom{n}{k} P\left(F_{2kr,2(n-k)r} > \frac{(n-k)t}{k(1-t)}\right)$

where $r \equiv$ gamma distribution shape parameter and $t \equiv$ test statistic value.

2.4 STATISTICAL OUTLIER MODELS IN INVENTORY THEORY

Early efforts in inventory theory to handle extreme value demands have many of the same shortcomings as statistical outlier theory. The following quote from Brown [4] underlies the basic assumptions:

> Occasionally there are demands that are recorded to be used in forecasting that should not be recorded. For example, there are keypunch errors in recording data, or demands that are really dependent demand, or demands that are to fill a large scheduled backlog type of order.

As in many statistical models, the implication is that an outlier is an error and an unusual occurrence. Inventory literature recommends the use of demand filters to identify large orders. Thus, any order that exceeds 3.5 standard deviations from the mean for tight control, or 4 standard deviations for normal control, or 5 standard deviations for loose control [4] should be earmarked to be checked for correctness. And if "...the demand is reasonable (for example, correct), it should be processed to increase the standard deviation" [4]. Although inventory scholars recognize that excluding extreme value demands reduces the variability of demand, and hence reduces the inventory investment in stock, inventory literature does not quantify the cost impact of applying filtering rules. In addition, there is no scientific evidence showing that a specified number of standard deviations from the mean is the most economical point for exclusion of extreme value demands.

3. A FILTERING RULE FOR LARGE INDIVIDUAL CUSTOMER ORDERS

For an empirical inventory systems study, Wagner [13] devised an outlier rule using successive observations of order statistics. We test a version of Wagner's rule for filtering out "large pop" customer orders. In this report, the rule is:

> Let $X_1, X_2, \ldots, X_k$ be the k largest observed customer orders out of N orders, where X_1 is the largest individual order and X_k the smallest. Given a value $r>1$, let $X_0=rX_1$ and define J as the set of j, for $1\le j\le k$, such that $X_{j-1}\ge rX_j$. Set $T_r=\min_{j\epsilon J}(rX_j)$.

We found in [1], [11] a statistical outlier rule by Likes similar to the rule above. Using the notation as above, Likes test statistic is $\frac{X_j - X_{j+1}}{X_j}$. Likes' [11] outlier model is applied sequentially and therefore suffers from the masking effect as does other classical outlier models. Wagner's filtering rule is not sequentially applied, and therefore is not subject to masking.

The parameters for the filtering rule are N, k, and r. To illustrate this method, we present three examples. Suppose N=25, k=10, and r=1.8. Assume the 10 largest observed orders for each example are:

SAMPLE CUSTOMER ORDERS

	EXAMPLE 1	EXAMPLE 2	EXAMPLE 3
X_1	29	1000	500
X_2	29	1000	500
X_3	28	800	300
X_4	27	100	100
X_5	23	28	25
X_6	20	27	20
X_7	19	27	13
X_8	18	25	7
X_9	17	23	7
X_{10}	17	20	6

TABLE 6

For example 1, X_0=52.2 (=1.8*29). The set J consists of all j, for $1 \le j \le 10$, such that $X_{j-1} \ge 1.8X_j$, which is j=1. Therefore, $T_{1.8}$=min(52.2)=52.2. Hence, any customer order that is at least 52.2 is filtered (specially handled); in the example, no previous order is considered a big pop.

For example 2, X_0=1800 (=1.8*1000). The set J consists of j=1, 4, and 5. Therefore, $T_{1.8}$=min (1800, 180, 50.4)=50.4. Hence, any order of at least 50.4 is filtered; in the example, the orders 1000, 1000, 800, and 100 are considered big pops.

For example 3, X_0=900 (=1.8*500). The set J consists of j=1, 4, 5, and 7. Therefore $T_{1.8}$=min (900, 180, 45, 12.6)=12.6. Example 3 illustrates the need for an amendment to the basic rule to prevent excessive filtering. In this example, the orders 25, 20, and 13 are indicated to be large pops. Therefore, we modify the rule by adding another parameter γ.

Given a value $\gamma>0$, let $w=\gamma(X_1-X_k)$. Define J as the set of j, for $1 \le j \le .20N$, such that $X_{j-1} \ge rX_j$ and for $.20N<j\le k$, such that $X_{j-1} \le rX_j$ and $X_j-X_{j+1}>w$. Set $T_r=\min_{j\varepsilon J}(rX_j)$.

Applying this modified rule to example 3 with γ=.2, set J consists of j=1, 4, and 5. We exclude j=7 since $X_7-X_8<w$ (6<98.4). Therefore, $T_{1.8}$=min (900, 180, 45)=45. Hence, any customer order of at least 45 is filtered.

The modification of this rule places an additional restriction to filtering out more than .20 of the sample order data. We use .20 since it seems a practical bound for special handling, and we show in [2] that increasing the probability of demand special handled beyond .15 tends to decrease the amount of cost savings.

4. EXPERIMENTAL DESIGN

4.1 CUSTOMER ORDER DISTRIBUTION

We test the filtering rule on a simulated distribution of individual orders that resembles actual customer order data. We generate proportion P of the order distribution as small orders, and the remaining proportion of 1-P as "large pops." The order distribution, shown in Figure 1, is described as follows. Let ϕ be the probability of a customer order size z, where

$$\phi_z = \begin{cases} \Phi(1.5) & z=1 \\ \Phi(z+.5) - \Phi(z-.5) & z=2,\ldots, B-1 \\ \Phi(b) - \Phi(B-.5) & z=B \\ \frac{1-P}{I} & z \in Z, \end{cases}$$

where

$$\begin{aligned} &\Phi(z)=\int_0^z \lambda e^{-\lambda z} dz, \\ &b = \frac{\ln(1-p)}{-\lambda}, \\ &B=\mathrm{int}(b+.5), \qquad (1) \\ &B^+ = \left\lceil \frac{B+.5}{10} \right\rceil * 10, \\ &Z = \{z \mid z=B^+(10)100(100)1000\}, \\ &I=\text{dimension of } Z. \end{aligned}$$

For example, let P=.95 and λ=.1. Then b=29.957, B=30, B^+=40, Z=40, 50,..., 100, 200,..., 1000, and I=16. We consider four distributions. All have λ=.1, and we let P be .95, .90, .85, and .80. The cumulative probability distribution is shown in Figure 2.

CUMULATIVE ORDER DISTRIBUTION

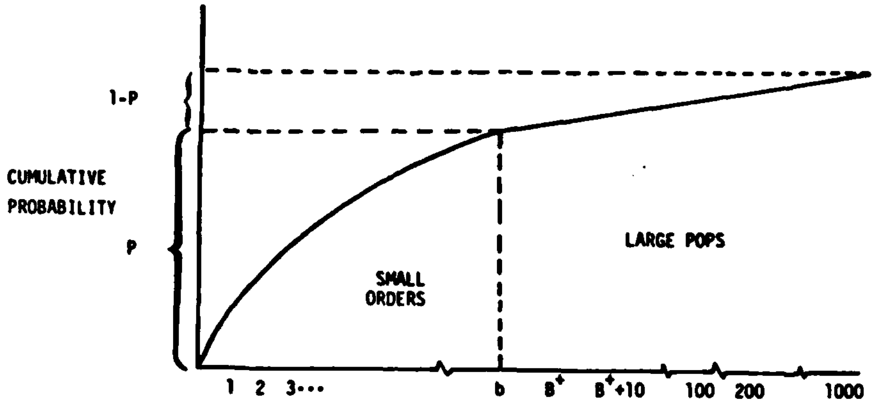

Figure 1

CUMULATIVE
ORDER DISTRIBUTION FOR
P=.95, λ=.1

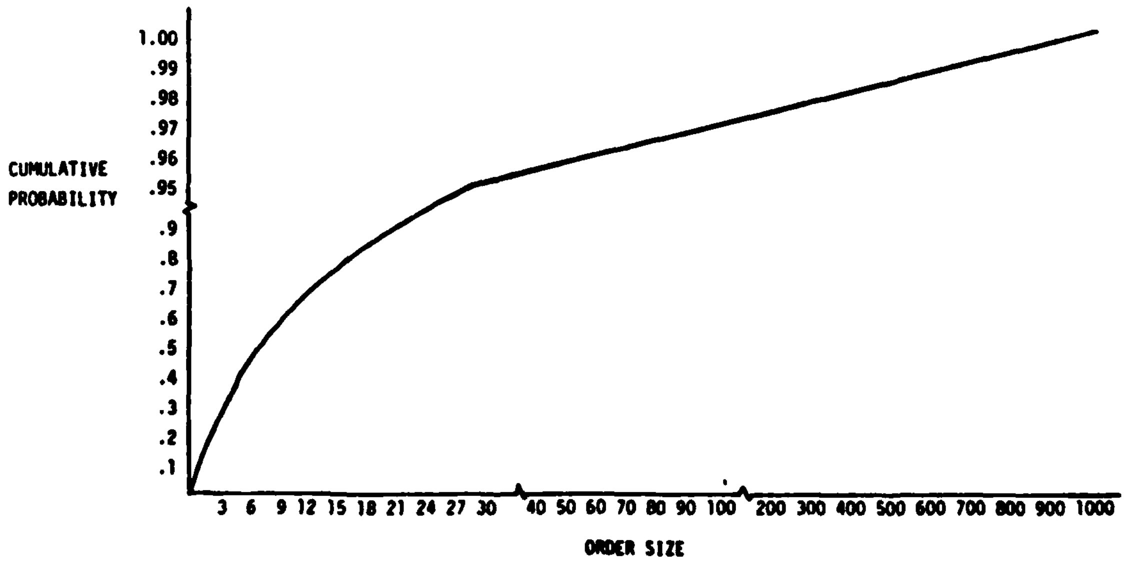

Figure 2

4.2 EXPERIMENTAL PROCESS

We generate N random customer orders (where N is the parameter for the filtering rule) and apply the filtering rule to find a value for T. We then generate another N random customer orders and find the associated value for T. For N=50, we generate 101 sets of orders and calculate the corresponding T for each set. For N=25, we generate 203 sets and values of T. This information is then formulated as shown in Table 7, which provides the observed distribution of the T values.

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-19	.0	-
20	.010	.140
33-39	.080	.052
44-49	.366	.050
50-54	.356	.046
72	.070	.041
90	.070	.038
107-162	.050	.029
≥200	.0	-

SIMULATION RESULTS FOR CASE 1

$P=.95$ $r=1.8$ $N=50$ $\gamma=.2$

TABLE 7

For the simulation appearing in Table 7, the first line shows that there is never a value of T between 1 and 19, and therefore customer orders of 19 or less are not filtered. The second line states that for 1% of the 101 sets of N=50, the T value was 20; therefore, all customer orders from 1 to 19 (86% of the orders) are considered ordinary

orders, and all orders from 20 to 1000 (14%) are filtered. The next line shows for 8% of the 101 sets, customer orders of 40 or more are filtered. This is .052 of the 50*101 simulated orders, which compares to the .05 for the hypothesized distiribution (see Figure 2). Note that the majority of the T values are in a limited range, with over 70% of the T values lying between 44 and 54. In fact, for all 24 of the cases found in Table 8, the observed distribution of the T values is concentrated in a narrow range (see the Appendix).

EXPERIMENTAL DESIGN

Parameter P	r	γ	N	k	Case Number
.95	1.8	.2,.01	50	15	1-2
	1.8	.2,.01	25	10	3-4
	1.6	.2	50	15	5
	1.6	.2	25	10	6
	1.99	.01	50	15	7
	1.99	.01	25	10	8
.90	1.8	.2,.01	50	15	9-10
	1.8	.2,.01	25	10	11-12
	1.6	.2	50	15	13
	1.6	.2	25	10	14
	1.99	.01	50	15	15
	1.99	.01	25	10	16
.85	1.8	.2	50	15	17
	1.8	.2	25	10	18
.80	1.8	.2,.01	50	15	19-20
	1.8	.2,.01	25	10	21-22
	1.99	.01	50	15	23
	1.99	.01	25	10	24

TABLE 8

4.3 OTHER CUSTOMER ORDER DISTRIBUTIONS

In order to test the sensitivity of the results to the form of the underlying customer order distribution, we also examine three other order distributions. We call the first order distribution, "linear small and linear large orders"; it is illustrated in Figure 3 and defined by

$$\phi_z = \begin{cases} \frac{.95}{30} & z=1,\ldots,30, \\ \frac{.05}{16} & z\varepsilon Z \end{cases}$$

where

$$Z=\{z \mid z=40(10)100(100)1000\}.$$

We use this distribution to test the sensitivity to increases in the frequency of the larger-valued small orders.

CUMULATIVE ORDER DISTRIBUTION FOR
LINEAR SMALL AND LINEAR LARGE
(P=.95)

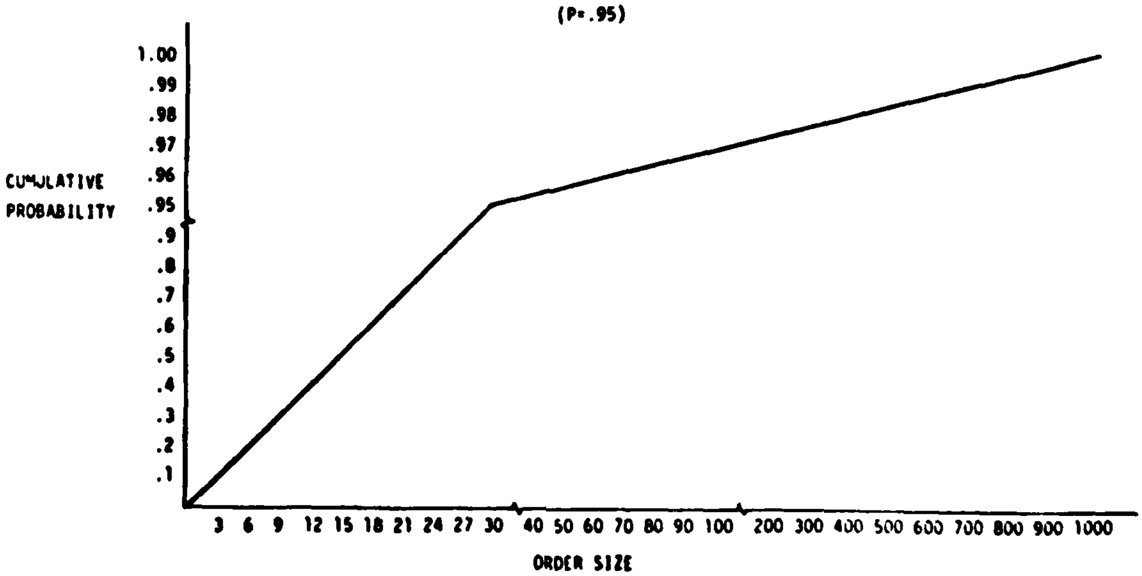

Figure 3

We call the second order distribution "exponential small and exponential large"; it is defined by

$$
\phi_z = \begin{cases} \Phi(1.5) & z=1 \\ \Phi(z+.5) - \Phi(z-.5) & z=2,\ldots,29 \\ \Phi(29.97) - \Phi(29.5) & z=30 \\ \Phi\left(30 + \frac{z-30}{10}\right) - \Phi\left(30 + \frac{z-40}{10}\right) & z=40,50,\ldots,100 \\ \Phi\left(30 + \frac{z-200}{100}\right) - \Phi\left(38 + \frac{z-300}{100}\right) & z=200,300,\ldots,900 \\ 1-\Phi(45) & z=1000, \end{cases}
$$

where

$$
\Phi(x)=\int_0^x \lambda e^{-\lambda x}dx.
$$

Thus, both small orders and large orders are distributed exponentially as shown in Figure 4. We employ this distribution to test the sensitivity to decreases in the frequency of larger-valued large pops.

CUMULATIVE ORDER DISTRIBUTION FOR
EXPONENTIAL SMALL AND EXPONENTIAL LARGE

(P=.95)

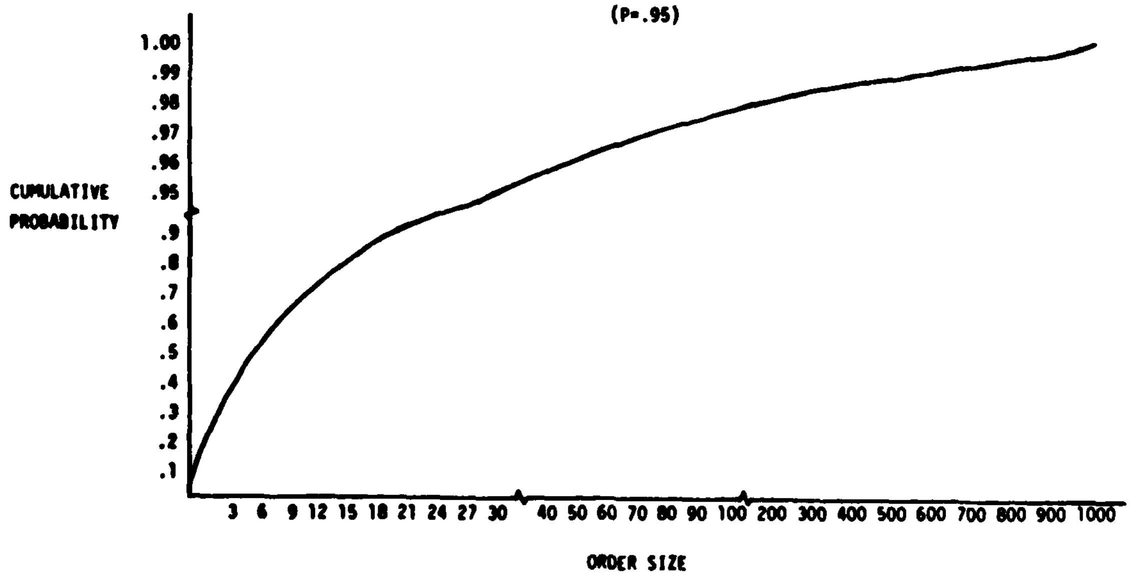

Figure 4

We call the third order distribution the "smaller average order"; it follows (1) with P=.90 and λ=.2 with two modifications. The first modification is to add a large order size of 150 and to reduce the range of the large order size from 1000 to 500. Thus, Z in (1) is changed to:

$$Z=\{z \mid z=6+(10)100,150,200(100)500\}.$$

A graph of this distribution is shown in Figure 5.

CUMULATIVE ORDER DISTRIBUTION FOR
SMALLER AVERAGE ORDER
WITHOUT ORDER SIZE OF 15

(P=.90)

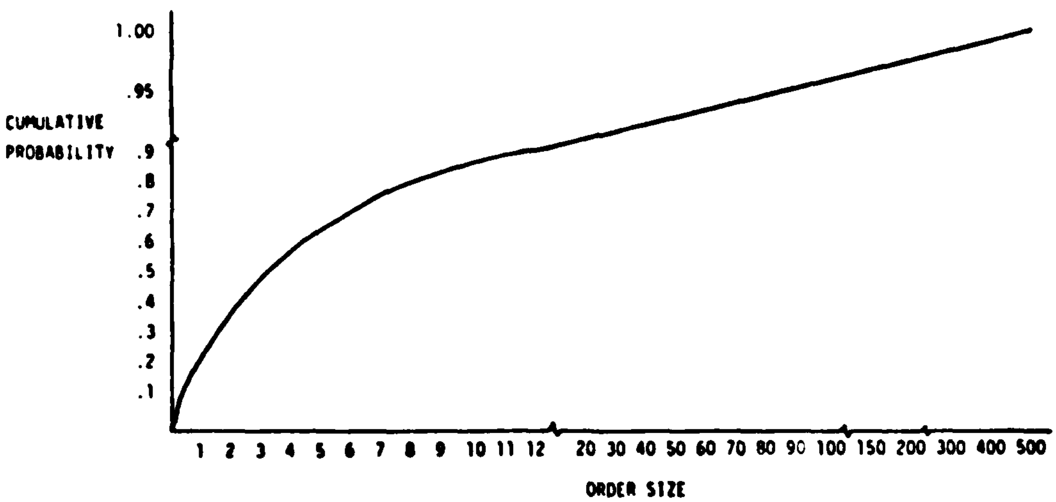

Figure 5

We use this distribution for three cases as shown in Table 9. The second modification is to add 15 to the large order sizes, thus Z in (1) is changed to:

$$Z=\{z \mid z=15,20(10)100,150,200(100)500\}.$$

We present the results of 2 cases with this distribution. A graph of this distribution is shown in Figure 6. We use this distribution to determine the effect of smaller order sizes and therefore larger order statistic ratios. The modifications to the distribution ensure that the ratio between any two successive order sizes is not greater than 1.8.

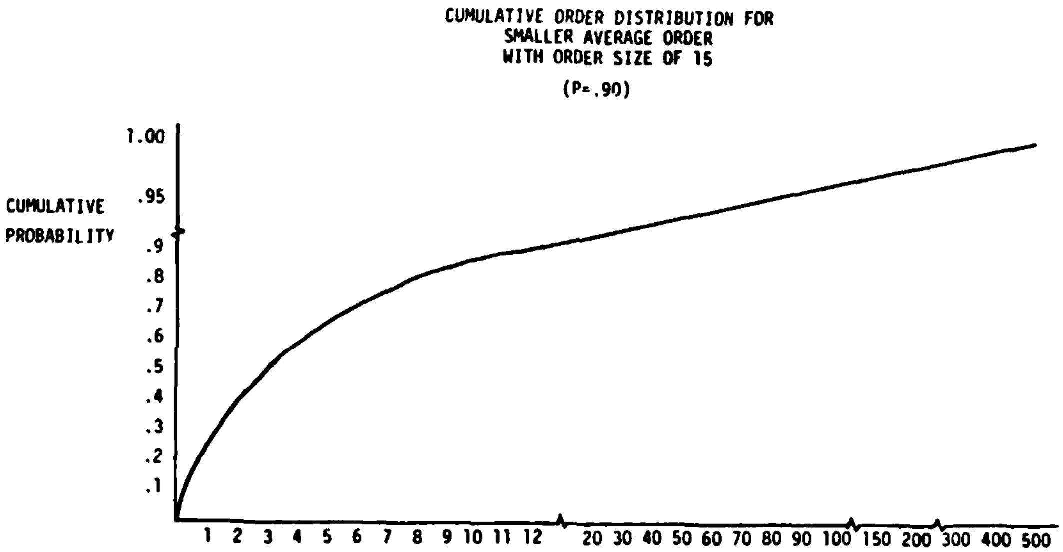

Figure 6

We use the experimental design shown in Table 9 for these other distributions.

EXPERIMENTAL DESIGN FOR

OTHER DISTRIBUTIONS

Distribution	P	r	γ	N	k	Case Number
Linear Small and Linear Large Orders	.95	1.8	.01	50	15	25
		1.8	.01	25	10	26
Exponential Small and Exponential Large Orders	.95	1.8	.01	50	15	27
		1.8	.01	25	10	28
Smaller Average Demand (λ=.2) (Without order size 15)	.90	1.8	.2,.01	50	15	29-30
		1.8	.2	25	10	31
(With order size 15)	.90	1.8	.01	50	15	32
		1.8	.01	25	10	33

TABLE 9

5. RESULTS

5.1 RESULTS WITH FILTERING RULE

For each value of P in Table 8 and each distribution in Table 9, we generate the same set of orders. Thus, there are seven distinct order sets: one for each of the four values of P listed in Table 8, and one for each of the three distributions listed in Table 9. We use the first N orders in each sequence to "initialize" the experiment. Use of the same order set allows us to compare the operating

characteristics for each of the parameter settings for the filtering rule.

Tables 10 through 13 summarize the results for the cases shown in Table 8.

SUMMARY OF RESULTS FOR P=.95 DISTRIBUTION

Case Number	r	γ	N	Average Percent of Orders Filtered	Variance of Percent of Orders Filtered	Relative Frequency of Filtering >1-P
1	1.8	.2	50	4.72	1.17	.010
2	1.8	.01	50	4.72	1.17	.010
3	1.8	.2	25	5.16	1.96	.055
4	1.8	.01	25	5.16	1.95	.055
5	1.6	.2	50	5.12	1.53	.020
6	1.6	.2	25	5.68	8.07	.193
7	1.99	.01	50	4.36	.33	.0
8	1.99	.01	25	4.72	.39	.010

TABLE 10

SUMMARY OF RESULTS FOR P=.90 DISTRIBUTION

Case Number	r	γ	N	Average Percent of Orders Filtered	Variance of Percent of Orders Filtered	Relative Frequency of Filtering 1-P
9	1.8	.2	50	7.93	1.29	.0
10	1.8	.01	50	7.93	1.29	.0
11	1.8	.2	25	8.95	3.30	.025
12	1.8	.01	25	8.96	3.26	.025
13	1.6	.2	50	8.92	.88	.010
14	1.6	.2	25	9.56	6.41	.060
15	1.99	.01	50	7.84	2.20	.0
16	1.99	.01	25	8.57	2.67	.015

TABLE 11

SUMMARY OF RESULTS FOR P=.85 DISTRIBUTION

Case Number	r	γ	N	Average Percent of Orders Filtered	Variance of Percent of Orders Filtered	Relative Frequency of Filtering >1-P
17	1.8	.2	50	11.52	3.22	.0
18	1.8	.2	25	12.08	3.85	.0

TABLE 12

SUMMARY OF RESULTS FOR P=.80 DISTRIBUTION

Case Number	r	γ	N	Average Percent of Orders Filtered	Variance of Percent of Orders Filtered	Relative Frequency of Filtering >1-P
19	1.8	.2	50	15.08	20.47	.0
20	1.8	.01	50	15.25	16.48	.0
21	1.8	.2	25	15.73	23.50	.01
22	1.8	.01	25	15.88	21.73	.01
23	1.99	.01	50	14.91	15.75	.0
24	1.99	.01	25	15.55	19.79	.0

TABLE 13

The tables identify the parameter settings (r, N, and γ), provide the average percent of orders filtered, and the variance of the percent of orders filtered. The tables also show the relative frequency that the rule filters out more than simulated (1-P) of the orders. This is analogous to a Type I error in the Quality Control sampling literature [6], [8].

For example, the first case in Table 10 has only 1% of the cases where more than 5.2% of the orders were designated to be filtered (see Table 7). Note we use the simulated value for 1-P, which as shown in Table 7 is .052.

Observe that for N=50, and P=.90 and .95, the filtering rule tends to be conservative; it filters slightly less than 1-P of the orders on average, with little variance and with little or no chance of a Type I error.

Increasing the fraction 1-P of large pops tends to increase the difference between 1-P and the average percent of orders filtered, and tends to increase the variance. The frequency of a Type I error, however, remains negligible.

Generally, we can draw the following conclusions regarding the parameters of the filtering rule:

1. As N or r increases, a smaller percentage of orders are filtered, the variance decreases, and there is less chance of a Type I error.
2. Type I error hardly varies as γ increases.

Based on the difference between 1-P and the percentage of orders filtered, the variance in the percentage of orders filtered, and the frequency of a Type I error, we recommend the use of the modified filtering rule with r=1.8 and γ=.2. In a practical setting the filtering rule would be applied to a period's worth of customer orders, say every 6 or 12 months. Therefore, N would vary from period to period. We recommend the period length be selected to include at least 25 orders.

We next examine the sensitivity of the filtering rule to changes in the order distribution. Tables 14 through 16 present a summary of the results for the cases shown in Table 9.

SUMMARY OF RESULTS FOR LINEAR SMALL AND LINEAR LARGE ORDERS (P=.95)

Case Number	r	Y	N	Average Percent of Orders Filtered	Variance of Percent of Orders Filtered	Relative Frequency of Filtering >1-P
25	1.8	.01	50	4.41	.32	.0
26	1.8	.01	25	4.60	.15	.0

TABLE 14

SUMMARY OF RESULTS FOR EXPONENTIAL SMALL AND EXPONENTIAL LARGE ORDERS (P=.95)

Case Number	r	Y	N	Average Percent of Orders Filtered	Variance of Percent of Orders Filtered	Relative Frequency of Filtering >1-P
27	1.8	.01	50	4.01	.44	.0
28	1.8	.01	25	4.89	4.41	.055

TABLE 15

The filtering rule performs equally well on these order distributions. In fact, for the smaller average order distributions, the rule performs better in terms of a smaller difference in the average fraction of customer orders filtered as compared to 1-P, and a smaller variance (see Table A-29). Note that the results for case 30, which uses the smaller average order distribution, are that 97% of the time

all large pops (orders of 20 or larger) are filtered. Whenever the smallest large pop value is at least r times as large as the largest small order, the filtering rule consistently filters the large pops, as we would expect.

SUMMARY OF RESULTS FOR SMALLER AVERAGE ORDERS (λ=.2, P=.90)

Case Number	r	γ	N	Average Percent of Orders Filtered	Variance of Percent of Orders Filtered	Relative Frequency of Filtering >1-P
29	1.8	.2	50	9.88	1.01	.0
30	1.8	.01	50	9.95	.11	.0
31	1.8	.2	25	10.07	3.30	.025
32	1.8	.01	50	9.01	.30	.0

TABLE 16

5.2 RESULTS WITH OTHER OUTLIER MODELS

We test two other outlier formulas to compare to our filter rule. The first rule is from [4] which states any order exceeding Y standard deviations from the mean should be identified for filtering. Using the distributions as used in cases 9, 10, 13, and 15 (P=.90), we generate the same 101 sets of 50 orders. We test the rule:

$$T = \text{sample mean} + Y * \text{sample standard deviation}.$$

We display the results for Y=2, 3 in Table 17.

SUMMARY OF RESULTS FOR
BROWN'S FILTERING RULE
(P=.90, N=50)

Y Value	Average Percent of Orders Filtered	Variance of Percent of Orders Filtered	Relative Frequency Of Filtering >1-P
2	4.28	1.67	.0
3	3.54	3.29	.0

TABLE 17

Neither of the two cases perform as well as our filtering rule in terms of the difference in the average proportion of demand filtered as compared to 1-P, and in terms of consistency.

We next test outlier model 5 from Tables 3 through 5, which is the model that perfomred the most reliably for the three hypothetical examples shown in Table 1. The procedure is:

Let $X_1,\ldots,X_k$ be the k largest observed customer orders, where X_1 is the largest and X_k the smallest. Determine the largest value of t, for t=1,2,3,4, such that

$$\frac{\sum_{j=1}^{t} X_t - t\bar{x}}{s} = F_t \geq C_t,$$

where $\bar{x}$=sample mean, s=sample standard deviation, and C_t=critical test statistic value found in [1]. If $F_t < C_t$ for all t, then $T=X_1+1$. Otherwise select the largest value of t where $F_t \geq C_t$, and set $T=X_t-1$.

We employ the distribution that we used for cases 3,4,6, and 8 (P=.95), which generates the same 203 sets of 25 orders. Note we

use P=.95 and N=25, because critical test statistical values are only available for t=4. The results are shown in Table 18. This rule does not perform as well as our rule, with the probability of a Type I error being .761. This rule is subject to swamping and is not a reliable performer for practical inventory applications.

SUMMARY OF RESULTS FOR
OUTLIER MODEL 5
(P=.95, N=25)

Average Percent of Orders Filtered	Variance of Percent of Orders Filtered	Relative Frequency of Filtering >1-P
12.03	67.98	.761

TABLE 18

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APPENDIX

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-19	.0	-
20	.010	.140
33-40	.080	.052
41-49	.366	.050
50-54	.356	.046
72	.070	.041
90	.070	.038
107-162	.050	.029
≥200	.0	-

SIMULATION RESULTS FOR CASE 2

$P = .95 \quad r = 1.8 \quad N = 50 \quad \gamma = .01$

TABLE A - 1

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-19	.0	-
20	.015	.140
22	.010	.118
26	.005	.079
27	.010	.071
29	.015	.061
31-38	.236	.053
41-49	.414	.050
51-54	.222	.046
72	.034	.041
90	.025	.038
108-126	.015	.030
≥200	.0	-

SIMULATION RESULTS FOR CASE 3

$P = .95 \quad r = 1.8 \quad N = 25 \quad \gamma = .2$

TABLE A - 2

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-19	.0	-
20	.015	.140
22	.010	.118
26	.005	.079
27	.010	.071
29	.015	.061
31-38	.236	.053
41-49	.419	.050
51-54	.217	.046
72	.034	.041
90	.025	.038
108-126	.015	.030
≥200	.0	-

SIMULATION RESULTS FOR CASE 4

$P = .95 \quad r = 1.8 \quad N = 25 \quad \gamma = .01$

TABLE A - 3

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-17	.0	-
18	.010	.169
29	.010	.061
31-40	.218	.052
42-48	.673	.050
64	.069	.044
96	.010	.034
144	.010	.029
≥200	.0	-

SIMULATION RESULTS FOR CASE 5

$P = .95 \quad r = 1.6 \quad N = 50 \quad \gamma = .2$

TABLE A - 4

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-14	.0	-
15	.005	.235
16	.005	.209
18	.015	.170
20	.025	.140
21	.015	.131
23	.010	.108
24	.015	.099
26	.020	.079
28	.034	.066
29	.049	.061
31-40	.374	.053
41-48	.339	.050
64	.034	.044
80	.005	.041
96	.005	.034
>100	.0	-

SIMULATION RESULTS FOR CASE 6

P = .95 r = 1.6 N = 25 γ = .2

TABLE A - 5

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-30	.0	-
36-38	.030	.052
42-50	.170	.050
52-60	.544	.045
80	.089	.041
100	.109	.034
120-180	.059	.029
>200	.0	-

SIMULATION RESULTS FOR CASE 7

P = .95 r = 1.09 N = 50 γ = .01

TABLE A - 6

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-23	.0	-
24	.005	.098
28	.005	.065
30	.005	.055
32-40	.138	.053
42-50	.325	.050
52-60	.409	.046
80	.050	.041
100	.044	.034
120-180	.020	.030
≥200	.0	-

SIMULATION RESULTS FOR CASE 8

$P = .95 \quad r = 1.99 \quad N = 25 \quad \gamma = .01$

TABLE A - 7

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-30	.0	-
31-40	.525	.092
42	.150	.086
54	.139	.082
72	.069	.067
90	.020	.060
108-180	.019	.048
.200	.0	-

SIMULATION RESULTS FOR CASE 9

$P = .90 \quad r = 1.8 \quad N = 50 \quad \gamma = .2$

TABLE A - 8

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-30	.0	-
31-40	.525	.092
42	.150	.086
54	.139	.082
72	.069	.067
90	.020	.060
108-180	.019	.048
≥200	.0	-

SIMULATION RESULTS FOR CASE 10

P = .90 r = 1.8 N = 50 γ = .01

TABLE A - 9

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-12	.0	-
13	.005	.284
20	.015	.139
22	.005	.112
27-29	.050	.097
31-40	.685	.091
42	.100	.086
54	.064	.081
72	.030	.064
90	.015	.060
107-180	.034	.048
≥200	.0	-

SIMULATION RESULTS FOR CASE 11

P = .90 r = 1.8 N = 25 γ = .2

TABLE A - 10

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-12	.0	-
13	.005	.284
20	.015	.139
22	.005	.112
27-29	.050	.097
31-40	.685	.091
42	.100	.086
54	.064	.081
72	.034	.064
90	.015	.060
107-180	.030	.048
≥200	.0	-

SIMULATION RESULTS FOR CASE 12

$P = .90 \quad r = 1.8 \quad N = 25 \quad \gamma = .01$

TABLE A - 11

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-20	.0	-
21	.010	.124
28-29	.069	.097
31-37	.663	.092
48	.168	.086
64	.059	.074
144-160	.030	.048
≥200	.0	-

SIMULATION RESULTS FOR CASE 13

$P = .90 \quad r = 1.6 \quad N = 50 \quad \gamma = .2$

TABLE A - 12

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-11	.0	-
12	.005	.313
16	.015	.210
18	.015	.171
20	.015	.139
21	.010	.123
24-39	.207	.097
31-37	.626	.091
48	.064	.086
64	.020	.073
128-160	.025	.048
.200	.0	-

SIMULATION RESULTS FOR CASE 14

$P = .90 \quad r = 1.6 \quad N = 25 \quad \gamma = .2$

TABLE A - 13

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-30	.0	-
34-40	.158	.092
42-46	.446	.096
60	.139	.082
80	.089	.067
100	.010	.055
120-199	.158	.048
200	.0	-

SIMULATION RESULTS FOR CASE 15

$P = .90 \quad r = 1.99 \quad N = 50 \quad \gamma = .01$

TABLE A - 14

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-13	.0	-
14	.005	.258
22	.010	.112
30	.015	.097
32-40	.399	.091
42-46	.379	.086
60	.074	.081
80	.060	.067
100	.020	.054
120-180	.039	.048
200	.0	-

SIMULATION RESULTS FOR CASE 16

$P = .90 \quad r = 1.99 \quad N = 25 \quad \gamma = .01$

TABLE A - 15

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-26	.0	-
27-29	.050	.136
31-36	.564	.126
54	.178	.112
72	.030	.099
90	.040	.092
125-180	.139	.077
≥200	.0	-

SIMULATION RESULTS FOR CASE 17

$P = .85 \quad r = 1.8 \quad N = 50 \quad \gamma = .2$

TABLE A - 16

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-19	.0	-
20	.005	.244
22-29	.230	.135
31-36	.560	.127
54	.090	.113
72	.020	.099
90	.010	.092
108-180	.060	.077
540	.005	.043
≥1000	.010	.0

SIMULATION RESULTS FOR CASE 18

$P = .85 \quad r = 1.8 \quad N = 25 \quad \gamma = .2$

TABLE A - 17

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-23	.0	-
24-29	.376	.190
36	.181	.180
54	.050	.152
90	.030	.119
107-180	.337	.102
1000	.020	.0

SIMULATION RESULTS FOR CASE 19

$P = .80 \quad r = 1.8 \quad N = 50 \quad \gamma = .2$

TABLE A - 18

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-23	.0	-
24-29	.376	.190
36	.188	.180
54	.040	.152
72	.010	.129
90	.059	.119
108-180	.317	.102
540	.010	.054
≥600	.0	-

SIMULATION RESULTS FOR CASE 20

$P = .80 \quad r = 1.8 \quad N = 50 \quad \gamma = .01$

TABLE A - 19

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-12	.0	-
13	.005	.280
15	.005	.231
17-20	.034	.200
22-29	.488	.190
36	.118	.180
54	.034	.152
72	.015	.129
90	.044	.119
107-180	.202	.101
360	.015	.080
540	.010	.054
720	.005	.032
1000	.025	.0

SIMULATION RESULTS FOR CASE 21

$P = .80 \quad r = 1.8 \quad N = 25 \quad \gamma = .2$

TABLE A - 20

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-12	.0	-
13	.005	.280
15	.005	.231
17-20	.034	.200
22-29	.486	.190
36	.118	.180
54	.030	.152
72	.015	.129
90	.054	.119
108-180	.202	.101
360	.015	.080
540	.010	.054
720	.010	.032
900	.005	[illegible]
>1000	.010	.0

SIMULATION RESULTS FOR CASE 22

P = .80   r = 1.[illegible]   N = 25   γ = .01

TABLE A - 21

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-25	.0	-
26-30	.208	.190
31-40	.327	.180
60	.059	.152
80	.010	.129
100	.050	.111
120-199	.337	.102
597	.010	.054
>600	.0	-

SIMULATION RESULTS FOR CASE 23

P = .80   r = 1.99   N = 50   γ = .01

TABLE A - 22

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-17	.0	-
18-20	.010	.200
21-30	.360	.190
31-40	.261	.180
60	.049	.152
80	.015	.129
100	.054	.111
120-199	.207	.102
398	.015	.080
597	.005	.054
796	.010	.032
995	.005	.010
≥1000	.010	.0

SIMULATION RESULTS FOR CASE 24

$P = .80 \quad r = 1.99 \quad N = 50 \quad \gamma = .01$

TABLE A - 23

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-41	.0	-
42	.010	.050
51-54	.723	.047
72	.119	.041
90	.030	.038
108-180	.119	.030
≥200	.0	-

SIMULATION RESULTS FOR CASE 25

LINEAR SMALL AND LINEAR LARGE ORDERS

$P = .95 \quad r = 1.8 \quad N = 50 \quad \gamma = .01$

TABLE A - 24

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-37	.0	-
38	.005	.053
42-49	.089	.050
51-54	.778	.047
72	.069	.041
90	.020	.038
126-180	.039	.030
≥200	.0	-

SIMULATION RESULTS FOR CASE 26

LINEAR SMALL AND LINEAR LARGE ORDERS
$P = .95 \quad r = 1.8 \quad N = 25 \quad \lambda = .01$

TABLE A - 25

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-37	.0	-
38-40	.050	.051
42-49	.356	.045
51-54	.307	.041
72	.129	.035
90	.040	.032
108-180	.119	.026
≥200	.0	-

SIMULATION RESULTS FOR CASE 27

EXPONENTIAL SMALL AND EXPONENTIAL LARGE ORDERS
$P = .95 \quad r = 1.8 \quad N = 50 \quad \lambda = .01$

TABLE A - 26

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-16	.0	-
17	.010	.194
18	.005	.174
20	.010	.143
26	.020	.079
27	.010	.070
29	.010	.057
31-38	.232	.051
40-49	.409	.045
51-54	.197	.042
72	.069	.035
90	.020	.032
108-144	.010	.026
≥200	.0	-

SIMULATION RESULTS FOR CASE 28

EXPONENTIAL SMALL AND EXPONENTIAL LARGE ORDERS
$P = .95 \quad r = 1.9 \quad N = 25 \quad \gamma = .01$

TABLE A - 27

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-14	.0	-
15-20	.970	.100
22	.010	.093
36	.010	.086
≥500	.010	.0

SIMULATION RESULTS FOR CASE 29

SMALLER AVERAGE ORDERS
$P = .90 \quad r = 1.8 \quad N = 50 \quad \gamma = .2$

TABLE A - 28

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-14	.0	-
15-20	.970	.100
22	.010	.093
36	.010	.086
54	.010	.071
≥100	.0	-

SIMULATION RESULTS FOR CASE 30

SMALLER AVERAGE ORDERS

P = .90 r = 1.8 N = 50 γ = .01

TABLE A - 29

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-7	.0	-
8	.005	.224
9	.020	.184
13-20	.941	.100
22	.005	.093
36	.005	.086
54	.005	.071
126	.005	.037
162	.010	.030
360	.005	.013
≥400	.0	-

SIMULATION RESULTS FOR CASE 31

SMALLER AVERAGE ORDERS

P = .90 r = 1.8 N = 25 γ = .2

TABLE A - 30

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-14	.0	-
15	.010	.100
17-20	.683	.094
22-27	.228	.086
36	.059	.079
54	.020	.065
≥60	.0	-

SIMULATION RESULTS FOR CASE 32

SMALLER AVERAGE ORDERS (WITH ORDER SIZE OF 15)
P = .90 r = 1.8 N = 25 γ = .01

TABLE A - 31

T Value	Relative Frequency of T Value	Cumulative Relative Frequency of Filtered Orders
1-7	.0	-
8	.005	.224
9	.020	.184
13-15	.094	.100
17-20	.719	.094
22-27	.113	.086
36	.025	.079
54	.005	.065
126	.005	.030
162	.010	.024
360	.005	.007
≥400	.0	-

SIMULATION RESULTS FOR CASE 33

SMALLER AVERAGE ORDERS (WITH ORDER SIZE OF 15)
P = .90 r = 1.8 N = 25 γ = .01

TABLE A - 32

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AD-A124 686 EVALUATION OF A 'LARGE POP' FILTERING RULE FOR INVENTORY MANAGEMENT SYSTEMS(U) NORTH CAROLINA UNIV AT CHAPEL HILL SCHOOL OF BUSINESS ADMINIS.. D BLAZER

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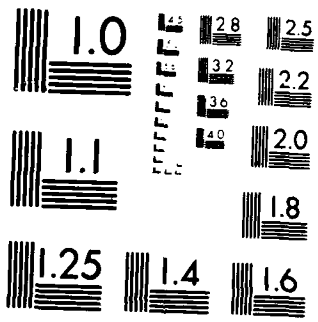

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SUPPLEMENTARY

INFORMATION

ERRATA

N00014-78-C-0467

AD. A124 686

Please note corrections to lines 11, 18 and 20 of Technical Report #22 page 15. These corrections appear on the following page.

For example 1, X_0=52.2 (=1.8*29). The set J consists of all j, for $1 \le j \le 10$, such that $X_{j-1} \ge 1.8X_j$, which is j=1. Therefore, $T_{1.8}$=min(52.2)=52.2. Hence, any customer order that is at least 52.2 is filtered (specially handled); in the example, no previous order is considered a big pop.

For example 2, X_0=1800 (=1.8*1000). The set J consists of j=1, 4, and 5. Therefore, $T_{1.8}$=min (1800, 180, 50.4)=50.4. Hence, any order of at least 50.4 is filtered; in the example, the orders 1000, 1000, 800, and 100 are considered big pops.

For example 3, X_0=900 (=1.8*500). The set J consists of j=1, 4, 5, and 8. Therefore $T_{1.8}$=min (900, 180, 45, 12.6)=12.6. Example 3 illustrates the need for an amendment to the basic rule to prevent excessive filtering. In this example, the orders 25, 20, and 13 are indicated to be large pops. Therefore, we modify the rule by adding another parameter γ.

Given a value $\gamma>0$, let $w=\gamma(X_1-X_k)$. Define J as the set of j, for $1 \le j \le .20N$, such that $X_{j-1} \ge rX_j$ and for $.20N<j \le k$, such that $X_{j-1} \le rX_j$ and $X_{j-1}X_j > w$. Set $T_r = \min_{j \varepsilon J}(rX_j)$.

Applying this modified rule to example 3 with γ=.2, set J consists of j=1, 4, and 5. We exclude j=8 since $X_7-X_8<w$ (6<98.4). Therefore, $T_{1.8}$=min (900, 180, 45)=45. Hence, any customer order of at least 45 is filtered.

The modification of this rule places an additional restriction to filtering out more than .20 of the sample order data. We use .20 since it seems a practical bound for special handling, and we show in [2] that increasing the probability of demand special handled beyond .15 tends to decrease the amount of cost savings.

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ILME